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Problem Set #2

2. Use the processes we have noted to write down an algorithm for solving the Rubiks Cube.

Solving the Rubiks Cube:

1. Put all eight corner cubelets in their correct positions and orientations.
 - (a) Position and orient the top corner cubelets using the following processes:
 - i. First, get all the top corner cubelets into the top layer.
 - A. If none of the top corner cubelets are in the top layer, the two processes L^2 and R^2 will bring all four to the top layer.
 - B. If one of the top corner cubelets is in the top layer, and we designate this corner cubelet as flu , one, two or three applications of D will place two of the top corner cubelets in the right layer. Then, two applications of R will place these two cubelets in the top layer. One, two or three applications of D will position the remaining top corner cubelet directly beneath the remaining corner not filled by a top layer cubelet, which we have designated ulb . The process BDB^{-1} will place the top corner cubelet in the top layer.

- C. If two of the top corner cubelets are in the top layer, look at their position. Then look at the bottom layer. One or two applications of D will position one of the top corner cubelets directly below a corner not yet filled by a top corner cubelet. If we designate this unfilled corner as fur , the process FDF^{-1} will position the cubelet in the top layer. Now repeat this process for the remaining top corner cubelet.
 - D. If three of the top corner cubelets are in the top layer, position the remaining corner cubelet directly below the unfilled top corner. If we designate this unfilled corner as fur , the process FDF^{-1} will position this cubelet in the top layer.
- ii. Position the top corner cubelets correctly using the following processes:
- A. If none of the top corner cubelets are in the correct positions, one or two applications of U will place at least one cubelet in the correct position.
 - B. If only one top corner cubelet is in the correct position, which we will designate ulb , one or two applications of $[F, R]U[R, F]U^{-1}$ will place either one or three of the remaining top corner cubelets in position. Continue on if necessary.
 - C. If two top corner cubelets are in their correct positions and share a layer, and we designate the correct cubelets as flu and ulb , one application of $[U, F]R^2[F, U]R^2$ will place the remaining top corner cubelets in the correct position.
 - D. If the two top corner cubelets are in their correct positions and are diagonally across from each other, the process $[F, R]U^2[R, F]U^2$ and one or two applications of U will place the top corner cubelets in their correct

positions.

iii. Orient the top corner cubelets using the following procedures:

- A. If none of the top corner cubelets are correctly oriented, one or two applications of $[F, U]^2[F, R]^2R^{-1}[R, D]^2R$ will orient two of the corners correctly. Then continue to part (C).
- B. If one of the top corner cubelets is correctly oriented, which we will designate as *ulb*, one or two applications of $[F, U]^2[F, R]^2R^{-1}[R, D]^2R$ will orient two more corners correctly. Then continue to part (E).
- C. If two of the top corner cubelets are correctly oriented and they share a layer, we will designate them as *ulb* and *ubr*. Then one or two applications of $[U, F]^2[U, L]^2L^{-1}[L, B]^2L$ will orient either one or two of the remaining corners. If one corner remains incorrectly oriented, continue to part (E).
- D. If two of the top corner cubelets are correctly oriented and they are diagonally across from each other, we will orient each remaining top corner cubelet separately. Continue to part (E) for this step and repeat for the second cubelet.
- E. If three cubelets are correctly oriented, we will designate the remaining cubelet as *flu*. Then one or two applications of the process $[F, R]^2R^{-1}[R, D]^2R$ will then orient it correctly.

(b) Position and orient the bottom corner cubelets.

i. Position the bottom corner cubelets correctly using the following processes:

- A. If none of the bottom corner cubelets are in the correct positions, one or two applications of D will place at

least one cubelet in the correct position.

- B. If only one bottom corner cubelet is in the correct position, which we will designate ldb , one or two applications of $[R, F]D[F, R]D^{-1}$ will place the remaining three in position.
 - C. If two bottom corner cubelets are in their correct positions and share a layer, with the correct two designated as lbd and brd , the process $[R, F]D[F, R]D^{-1}[R, F]D^2[F, R]D^2$ and one or two applications of D will place the bottom corner cubelets in their correct positions.
 - D. If the two bottom corner cubelets are in their correct positions and are diagonally across from each other, designated lbd and fdr , the process $[R, F]D[F, R]D^{-1}$ will place them in the same layer. Then return to part (C) to place them in their correct positions.
- ii. Orient the bottom corner cubelets using the following procedures:
- A. If none of the bottom corner cubelets are correctly oriented, one or two applications of $[R, D]^2[R, F]^2F^{-1}[F, U]^2F$ will orient two of the corners correctly. Then continue to part (C).
 - B. If one of the bottom corner cubelets is correctly oriented, which we will designate as lbd , one or two applications of $[R, D]^2[R, F]^2F^{-1}[F, U]^2F$ will orient one or three more corners correctly. Then continue to part (C) if needed.
 - C. If two of the bottom corner cubelets are correctly oriented and they share a layer, we will designate them as lbd and fld . Then one or two applications of $[D, L]^2L^{-1}[L, F]^2L$ will orient the remaining corners.

D. If two of the bottom corner cubelets are correctly oriented and they are diagonally across from each other, with one designated *lbd*, one application of $[R, F]D[F, R]D^{-1}$ will place the two incorrect cubelets in the same layer. Then return to part (C), follow the process, then repeat $[R, F]D[F, R]D^{-1}$ two more times to return the cubelets to their correct positions.

2. Put the top edge cubelets in their correct positions with the correct orientation.

(a) Place the top edge cubelets in top layer.

i. If any top edge cubelet is in the middle layer and is designated *fr* and *uf* is not filled with a top edge cubelet, the process $(F^2U^2)^3(L^{-1}FL)(F^2U^2)^3(L^{-1}F^{-1}L)$ will put the cubelet in the top layer. If *uf* is filled but *ur* is unfilled, two applications of $(R^2U^2)^3(F^{-1}RF)(R^2U^2)^3(F^{-1}R^{-1}F)$ will put the top edge cubelet in the top layer. Repeat for any other cubelets located in the middle layer, unless the *uf* and *ur* cubelets are both top edge cubelets. If this occurs, and the middle edge cubelet is still designated as *fr*, the process $L^{-1}D^{-1}(F^2D^2)^3DL$ will place the cubelet in the bottom row. Then continue to part (ii).

ii. If any top edge cubelet is in the bottom layer and is directly below an unfilled top edge position, we designate this top edge cubelet as *fd*. Then, the process $(DU^{-1}F)(R^2F^2)^3(F^{-1}UD^{-1})$ will put the cubelet in the top layer.

iii. If any top edge cubelet is in the bottom layer and is directly below another top edge cubelet, one of two processes will position it in the top layer:

A. If the top edge cubelet in the bottom layer is designated *fd* and the *ur* position is not filled with a top edge cubelet, the process $UR(F^2R^2)^3R^{-1}U^{-1}$ will position

the top edge cubelet in the top layer. If the ur position is filled, but the ul position is unfilled, the process $D^{-1}F^{-1}(L^2F^2)^3FD$ will position the top edge cubelet in the top layer.

B. If the top edge cubelet in the bottom layer is designated fd and both the ur and ul positions are filled with top edge cubelets, then the process $(FB^{-1}D)(L^2D^2)^3(D^{-1}BF^{-1})$ will position it directly below the open position. Return to part (ii) to place it in the top layer.

(b) Position the four top edge cubelets correctly in the top layer.

i. If all four top edge cubelets are in the wrong positions, the following processes will position them correctly:

A. If each edge cubelet is directly opposite its correct position, one application of $(FB^{-1}U)(R^2U^2)^3(U^{-1}BF^{-1})$ will put them in position.

B. If there are two pairs of cubelets that need to be switched, with one pair being uf - ur , one application of $(R^2D^2B^2D)(F^2L^2)^3(D^{-1}B^{-2}D^{-2}R^{-2})$ will put them in position.

C. If neither part (A) nor (B) applies, perform one application of $(U^2R^2)^3(B^{-1}UB)(U^2R^2)^3(B^{-1}U^{-1}B)$ and then return to part (A) or (B) as appropriate.

D. If three of the top edge cubelets are in the wrong positions, with the correct edge cubelet being designated ul , one or two applications of $(U^2R^2)^3(B^{-1}UB)(U^2R^2)^3(B^{-1}U^{-1}B)$ will position them correctly.

ii. If two of the top edge cubelets are out of position and they are directly opposite each other, with one designated uf , the process $(U^2L^2)^3(L^2D^2)^3$ will put them in position.

iii. If two of the top edge cubelets are out of position and they

are adjacent to each other, designated uf and ur , one application of $(FU^{-1}B^{-1}D)(R^2D^2)^3(D^{-1}BUF^{-1})(L^{-1}UL)(F^2D^2)^3(L^{-1}U^{-1}L)$ will place the uf cubelet in the correct ur position. Then return to part (2,a,i) to place the remaining edge cubelet back in the top layer in the correct position.

(c) Orient the top edge cubelets correctly.

- i. If a top edge cubelet, designated uf , is oriented incorrectly, one application of $(F^2R^2)^3(RD^{-1}RFL^{-1}F)(F^2R^2)^3(F^{-1}LF^{-1}R^{-1}DR^{-1})$ will orient it correctly. Repeat this step for any other incorrect top edge cubelets.

3. Position and orient the middle edge cubelets correctly.

(a) Place all the middle edge cubelets in the middle layer.

- i. If a middle edge cubelet is in the bottom layer, designated fd , and either the fr or fl positions is not filled with a middle edge cubelet, then one or two applications of $(F^2D^2)^3(R^{-1}FR)(F^2D^2)^3(R^{-1}F^{-1}R)$ will place the cubelet in the middle layer, and will keep either the fr or the fl cubelet in the middle layer, if either is also a middle edge cubelet.
- ii. If a middle edge cubelet is in the bottom layer, designated fd , and both the fr and fl positions are filled with middle edge cubelets, then the process $F^{-1}D^{-1}(R^2D^2)^3DF$ will place the cubelet in the rb position in the middle layer. If the rb position is already filled with a middle edge cubelet, then the process $FD(L^2D^2)^3D^{-1}F^{-1}$ will put the cubelet in the lb position in the middle layer.

(b) Position the middle edge cubelets correctly.

- i. If all four of the middle edge cubelets are positioned incorrectly, with each directly across from its correct

position, continue to part (iii) and repeat as necessary with each pair. If they are not all directly across from their correct positions, continue to part (ii) and follow the same process.

- ii. If three of the middle edge cubelets are positioned incorrectly, with their positions designated as lb , fl , and fr , continue to part (iii) and repeat once with the $lb-fl$ pair, and then again with the new $fl-fr$ pair. If necessary, repeat once more with the new $lb-fl$ pair.
- iii. If two of the middle edge cubelets are positioned incorrectly and are opposite one another, designated fl and fr , the process $(F^2D^2)^3$ will position them correctly.
- iv. If two of the middle edge cubelets are positioned incorrectly and are diagonally across from one another, designated fr and lb , first perform the process $L^{-1}D^{-1}(F^2D^2)^3DL$ to get the fr cubelet into the bottom left layer. Then apply the process $FD(L^2D^2)^3D^{-1}F^{-1}$ to get the lb cubelet into the bottom front layer. Then return to part (3,a,i) to get both of these cubelets back into the middle layer in their correct positions.

(c) Orient the middle edge cubelets correctly.

- i. If all four middle edge cubelets are oriented incorrectly, continue to part (iii) to correct them.
- ii. If three middle edge cubelets are oriented incorrectly, designated lb , fl , and fr , continue to part (iii) and repeat with the pair $lb-fl$ and then again with the new pair $fl-fr$. Then continue to part (v).
- iii. If two middle edge cubelets are oriented incorrectly and are directly opposite one another, designated fl and fr , the process $(F^2D^2)^3(DL^{-1}DFU^{-1}F)(F^2D^2)^3(F^{-1}UF^{-1}D^{-1}LD^{-1})$ will orient them correctly.

- iv. If two middle edge cubelets are oriented incorrectly and are diagonally across from one another, designated lb and fr , return to part (iii) and repeat with the pair $lb-fl$ and then again with the new pair $fl-fr$.
- v. If one middle edge cubelet is oriented incorrectly, designated fr , the process $(DBU(R^2U^2)^3U^{-1}B^{-1}D^{-1})(LD^{-1}L^{-1}(U^2F^2)^3LDL^{-1})$ will orient the cubelet correctly, though it will place it back in the bottom layer. Return to and perform part (3,a,i) to get all four middle edge cubelets back in the middle layer, and then reposition as necessary. The orientation will remain correct.

4. Position and orient the bottom edge cubelets correctly.

- (a) Position the bottom edge cubelets correctly (By this time, they should already be in the bottom layer!)
 - i. If all four bottom edge cubelets are in the wrong positions, the following processes will position them correctly:
 - A. If each edge cubelet is directly opposite its correct position, one application of $(FB^{-1}D)(L^2D^2)^3(D^{-1}BF^{-1})$ will put them in position.
 - B. If there are two pairs of cubelets that need to be switched, with one pair being $df-dl$, one application of $(L^2U^2B^2U)(F^2R^2)^3(U^{-1}B^{-2}U^{-2}L^{-2})$ will put them in position.
 - C. If neither part (A) nor (B) applies, perform one application of $(D^2L^2)^3(B^{-1}DB)(D^2L^2)^3(B^{-1}D^{-1}B)$ and then return to part (A) or (B) as appropriate.
 - ii. If three of the bottom edge cubelets are in the wrong positions, with the correct edge cubelet being designated dr , one or two applications of $(D^2L^2)^3(B^{-1}DB)(D^2L^2)^3(B^{-1}D^{-1}B)$ will position them correctly.

(b) Orient the bottom edge cubelets correctly.

- i. If two bottom edge cubelets are oriented incorrectly and are directly across from each other, designated df and db , one application of

$$(D^2R^2)^3(RB^{-1}RDL^{-1}D)(D^2R^2)^3(D^{-1}LD^{-1}R^{-1}BR^{-1})$$

will orient them correctly.

- ii. If two bottom edge cubelets are oriented incorrectly and are adjacent to each other, designated dr and db , the process

$$(D^2B^2)^3(R^{-1}DR)(D^2B^2)^3(R^{-1}D^{-1}R)$$

will place them directly opposite one another. Return to part (i) to orient them correctly, and then return to part (4,a,ii) to reposition them correctly. Congratulations! You've just solved the Rubiks Cube!